Cheat sheet on the simplicial indexing category

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The category Δ has objects $[n]$ for $n \geq 0$ and its morphisms are generated by

(1) $\mathrm{id}_n \colon [n] \to [n],$

and

(2) $\partial_j^n \colon [n-1] \to [n],$
     $\sigma_j^n \colon [n+1] \to [n]$

for $j = 0, \dots, n$, subject to the relations

(3) $\partial_j^{n+1} \circ \partial_i^n = \partial_i^{n+1} \circ \partial_{j-1}^n \colon [n-1] \to [n+1]$

for $0 \leq i < j \leq n+1$,

(4) $\sigma_j^n \circ \sigma_i^{n+1} = \sigma_i^n \circ \sigma_{j+1}^{n+1} \colon [n+2] \to [n]$

for $0 \leq i \leq j \leq n$, and

(5) $\sigma_j^{n-1} \circ \partial_i^n = \begin{cases} \partial_i^{n-1} \circ \sigma_{j-1}^{n-2} & i < j \\ \mathrm{id}_{n-1} & i = j, j+1 \\ \partial_{i-1}^{n-1} \circ \sigma_j^{n-2} & j+1 < i \end{cases}$

for $0 \leq i, j \leq n$.