Cheat sheet on mutations

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Summary

Let \((E, F)\) be an exceptional pair. Then
1. \((L_E F, E)\) is the left mutation, the mutated object is on the left;
2. \((F, R_F E)\) is the right mutation, the mutated object is on the right.

The corresponding triangles are

\[
\begin{align*}
L_E F & \rightarrow \bigoplus_{n \in \mathbb{Z}} \text{Hom}(E, F[n]) \otimes_k E[-n] \xrightarrow{\text{can}} F \rightarrow L_E F[1] \\
E & \xrightarrow{\text{can}^*} \bigoplus_{n \in \mathbb{Z}} \text{Hom}(E, F[n])^\vee \otimes_k F[n] \xrightarrow{\text{can}} R_F E \rightarrow E[1].
\end{align*}
\]

Consider \(\mathcal{T}\) a triangulated category over a field \(k\). Let \((E, F)\) be an exceptional pair in \(\mathcal{T}\). The natural evaluation maps

\[
(2) \quad \text{ev}: \text{Hom}(E[-n], F) \otimes_k E[-n] \rightarrow F
\]

resp. coevaluation maps

\[
(3) \quad \text{ev}^*: E \rightarrow \text{Hom}(E, F[n])^\vee \otimes_k F[n]
\]

can be assembled to the canonical map

\[
(4) \quad \text{can}: \bigoplus_{n \in \mathbb{Z}} \text{Hom}(E, F[n]) \otimes_k E[-n] \rightarrow F
\]

resp. the dual canonical map

\[
(5) \quad \text{can}^*: E \rightarrow \bigoplus_{n \in \mathbb{Z}} \text{Hom}(E, F[n]) \otimes_k F[n].
\]

Definition 1.

The left mutation of \((E, F)\) is the exceptional pair \((L_E F, E)\), where \(L_E F[1]\) is the cone of \(\text{can}\).

The right mutation of \((E, F)\) is the exceptional pair \((F, R_F E)\), where \(R_F E\) is the cone of \(\text{can}^*\).