Representation Theory I
Bonn, summer term 2019

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Sheet 7

Solutions to be handed in before class on Wednesday May 22

There is the following remark about last week's lecture:

> In the argument that the coroot is unique one needs that in the last step the field has characteristic zero, since n should not be zero.

Problem 34. Let V be a finite-dimensional vector space over a field of characteristic 0. Let $R \subseteq V$ be an irreducible root system. Let W be the associated Weyl group. Show that V is an irreducible representation of W. (3 points)

Problem 35. Recall that the *Cartan matrix* of a root system is the matrix $(a_{j,i}) = (2(\alpha_i, \alpha_j)/(\alpha_j, \alpha_j))_{i,j}$, where $\alpha_1, \ldots, \alpha_n$ are the simple roots.

Recall that the *Dynkin diagram* of a root system is a visual representation of the Cartan matrix. It is a graph whose vertices correspond to the simple roots. An undirected single edge is drawn if the angle of the roots is 60° or 120°. A directed double edge is drawn if the angle is 45° or 135°, oriented from the longer to the shorter root, and likewise a directed triple edge is drawn if the angle is 30° or 150°.

1. Show that up to isomorphism there are 4 rank 2 root systems, and draw their pictures. (4 points)
2. Compute their Cartan matrices, and draw their associated Dynkin diagrams. (4 points)
3. Which of these are Langlands dual or self-dual? (2 points)

Problem 36. The root system of type B_2 is constructed in the previous exercise, and is the one involving double (and not single or triple) edges. If you haven't constructed the root system in the previous exercise, take a look at `https://en.wikipedia.org/wiki/Root_system#/media/File:Root_system_B2.svg`.

Compute the order of the Weyl group of type B_2, and show which familiar group it is isomorphic to. (3 points)