Prof. Dr. Catharina Stroppel
Dr. Pieter Belmans

Sheet 9

Solutions to be handed in before class on Friday December 13.

Problem 40. Let C be an irreducible curve (i.e. a closed subvariety of dimension 1) of degree d (i.e. given by a homogeneous polynomial of degree d) in $\mathbb{P}^2$.

1. Prove that $[C] = d[\Omega_{s_1}]$.
2. Consider the excision sequence. Explain how it is compatible with the grading.
3. Determine the Chow group $\mathrm{A}^1(\mathbb{P}^2 \setminus C)$.
4. Determine the Chow group $\mathrm{A}^i(\mathbb{P}^n \setminus X)$ for $i = 0, \ldots, n - m$, where X is a subvariety of dimension $m < n$ and degree d.

(5 points)

Problem 41. Consider the Chow ring for $\mathbb{P}^1 \times \mathbb{P}^1$. Recall that curves on $\mathbb{P}^1 \times \mathbb{P}^1$ are described using bihomogeneous polynomials in x_1, x_2, y_1, y_2, i.e. they are homogeneous in x_1, x_2 and homogeneous (of possibly different degree) in y_1, y_2.

1. Prove that $\mathrm{A}^2(\mathbb{P}^1 \times \mathbb{P}^1) \cong \mathbb{Z}$, i.e. that all points are rationally equivalent to a given point (assuming that the class of a point is nonzero).

   (2 points)

2. Using that $\mathrm{A}^1(\mathbb{P}^1)$ is naturally isomorphic to $\mathrm{A}^1(\mathbb{P}^1 \times \mathbb{A}^1)$, prove that

$$\mathrm{A}^1(\mathbb{P}^1 \times \mathbb{P}^1) \cong \mathbb{Z} \oplus \mathbb{Z}. \tag{23}$$

   (3 points)

3. Construct a cell decomposition for $\mathbb{P}^1 \times \mathbb{P}^1$, and give an alternative proof of points 1 and 2.

   (2 points)

Problem 42. Continuing with the setup of Problem 41 we wish to compute the ring structure of the Chow ring $\mathrm{A}^\bullet(\mathbb{P}^1 \times \mathbb{P}^1)$.

1. Consider the subvarieties $C \coloneqq \{\mathrm{pt}\} \times \mathbb{P}^1$ and $D \coloneqq \mathbb{P}^1 \times \{\mathrm{pt}\}$. Compute the intersection products C^2 and CD.

   (2 points)

2. Let C (resp. D) be a curve of bidegree (c_1, c_2) (resp. (d_1, d_2)) under the isomorphism (23). Determine CD.

   (2 points)