Cheat sheet on mutations

Pieter Belmans

Summary

Let (E, F) be an exceptional pair. Then

1. $(\mathrm{L}_E\, F, E)$ is the *left mutation*, the mutated object is on the left;
2. $(F, \mathrm{R}_F\, E)$ is the *right mutation*, the mutated object is on the right.

The corresponding triangles are

$$\begin{aligned} &\mathrm{L}_E\, F \longrightarrow \bigoplus_{n\in\mathbb{Z}} \mathrm{Hom}(E, F[n]) \otimes_k E[-n] \xrightarrow{\mathrm{can}} F \longrightarrow \mathrm{L}_E\, F[1] \\ &E \xrightarrow{\mathrm{can}^*} \bigoplus_{n\in\mathbb{Z}} \mathrm{Hom}(E, F[n])^\vee \otimes_k F[n] \xrightarrow{\mathrm{can}} \mathrm{R}_F\, E \longrightarrow E[1]. \end{aligned} \tag{1}$$

Consider $\mathcal{T}$ a triangulated category over a field k. Let (E, F) be an exceptional pair in $\mathcal{T}$. The natural evaluation maps

$$\mathrm{ev}\colon \mathrm{Hom}(E[-n], F) \otimes_k E[-n] \to F \tag{2}$$

resp. coevaluation maps

$$\mathrm{ev}^*\colon E \to \mathrm{Hom}(E, F[n])^\vee \otimes_k F[n] \tag{3}$$

can be assembled to the *canonical map*

$$\mathrm{can}\colon \bigoplus_{n\in\mathbb{Z}} \mathrm{Hom}(E, F[n]) \otimes_k E[-n] \to F \tag{4}$$

resp. the *dual canonical map*

$$\mathrm{can}^*\colon E \to \bigoplus_{n\in\mathbb{Z}} \mathrm{Hom}(E, F[n]) \otimes_k F[n]. \tag{5}$$

Definition 1.

The *left mutation* of (E, F) is the exceptional pair $(\mathrm{L}_E\, F, E)$, where $\mathrm{L}_E\, F[1]$ is the cone of can.

The *right mutation* of (E, F) is the exceptional pair $(F, \mathrm{R}_F\, E)$, where $\mathrm{R}_F\, E$ is the cone of can^*.